

ECE 260 course notes

Every module's technical content in one place, by topic: the power system, AC circuits and three-phase power, magnetic circuits, transformers, DC machines, synchronous machines and induction motors – formulas, the highest-value worked examples, and practice with answers.

The power system

Generation at 13.8–24 kV, transmission meshed for reliability at 230–765 kV, sub-transmission at 69–138 kV, distribution radial for cost at 2.2–46 kV down to a service voltage of 120–480 V.

Phasors, impedance, and instantaneous power

Module 2 part A: the circuit language every later module is written in.

A lowercase symbol is instantaneous, a capital is RMS.

KEY RELATION

$$v(t) = \sqrt{2} V \cos \omega t, \quad i(t) = \sqrt{2} I \cos(\omega t - \phi), \quad V = \frac{V_{max}}{\sqrt{2}}; \quad \bar{V} = V \angle 0^\circ, \quad \bar{I} = I e^{-j\phi} = I \angle -\phi.$$

A lagging current has a negative angle: $\phi > 0$ puts $\bar{I}$ below the real axis.

KEY RELATION

$$Z = \frac{V \angle \theta_v}{I \angle \theta_i} = |Z| \angle \theta, \quad j\omega L = jX_L, \quad -j\frac{1}{\omega C} = -jX_C, \quad Z_{RLC} = R + j(X_L - X_C).$$

Instantaneous power splits into a part that never goes negative and a part that averages to zero:

KEY RELATION

$$p(t) = P(1 + \cos 2\omega t) + Q \sin 2\omega t, \quad P = VI \cos \phi \text{ [W]}, \quad Q = VI \sin \phi \text{ [VAR]}.$$

P is never negative; Q averages exactly zero – a pure L or C borrows energy and returns it each cycle.

KEY RELATION

$$PF = \cos \phi, \quad RF = \sin \phi, \quad \phi > 0 \text{ for lagging current.}$$

Lagging is inductive, leading capacitive; always quote which.

Complex power and power-factor correction

Module 2 part B.

KEY RELATION

$$S = \bar{V}\bar{I}^* = VIe^{j\phi} = VI(\cos \phi + j \sin \phi) = P + jQ \text{ [VA]},$$

$$|S| = VI, \quad P = |S| \cos \phi, \quad Q = |S| \sin \phi.$$

$Q > 0$ (current lags) means the circuit absorbs reactive power, inductive; $Q < 0$ (current leads) means it supplies reactive power, capacitive.

For a series branch $Z = R + jX$:

KEY RELATION

$$P = I^2 R, \quad Q = I^2 X, \quad \frac{Q}{P} = \frac{X}{R} = \tan \phi, \quad S = Z I^2.$$

A parallel capacitor across the load voltage V supplies $Q_c = V^2/X_C$, subtracting from what the supply must provide:

KEY RELATION

$$Q_{new} = Q_{old} - Q_c, \quad P_{new} = P, \quad X_C = \frac{V^2}{Q_c}, \quad C = \frac{1}{\omega X_C} = \frac{Q_c}{\omega V^2}.$$

The base P never changes; $|S|$ and the supply current shrink. Past $Q_c = Q_{old}$ the angle crosses to leading – over-correction is not free.

WORKED EXAMPLE · MODULE 2, EXAMPLE 2 (P.27) · CORE · NUMBERS FROM P.28 OF THE SOLVED DECK

For the circuit shown, calculate Q_{old} and C for $PF_{new} = 1$. The load is $Z = 11 + j11 = 15.56\angle 45^\circ$, the supply is $V = 220 \text{ V}$, and the capacitor C is in parallel with Z .

Step 1, the load current:

$$\bar{I}_Z = \frac{\bar{V}}{Z} = \frac{220\angle 0^\circ}{15.56\angle 45^\circ} = 14.14\angle -45^\circ \text{ A.}$$

Step 2, the reactive power to cancel:

$$Q_{old} = VI_Z \sin \phi_{old} = 220 \times 14.14 \times \sin 45^\circ = 2200 \text{ VAR} = I^2 X = 14.14^2 \times 11.$$

Step 3, the capacitor. Unity power factor means $Q_c = Q_{old} = 2200 \text{ VAR}$, so

$$X_C = \frac{V^2}{Q_c} = \frac{220^2}{2200} = 22 \Omega \Rightarrow C = \frac{1}{\omega X_C} = \frac{1}{2\pi(60)(22)} = 120.6 \mu\text{F}.$$

Check. $\bar{I}_C = 220/(22\angle -90^\circ) = 10\angle +90^\circ \text{ A}$, and

$$\bar{I} = \bar{I}_Z + \bar{I}_C = (10 - j10) + (0 + j10) = 10\angle 0^\circ \text{ A},$$

in phase with the voltage, so the power factor is 1.

Balanced three-phase circuits

Module 2 part C.

KEY RELATION

$$\bar{V}_{an} = V\angle 0^\circ, \quad \bar{V}_{bn} = V\angle -120^\circ, \quad \bar{V}_{cn} = V\angle -240^\circ.$$

In a **star** (Y), line voltages are differences of phase voltages. In a **delta** the phase voltages *are* the line voltages, and the phase currents' differences give the line currents:

KEY RELATION

$$\bar{V}_{ab} = \sqrt{3} \bar{V}_a \angle 30^\circ, \quad \bar{I}_a = \bar{I}_{ab} - \bar{I}_{ca} = \sqrt{3} \bar{I}_{ab} \angle -30^\circ.$$

STAR AND DELTA, SIDE BY SIDE

	STAR (Y)	DELTA
Voltages	$V_L = \sqrt{3} V_{ph}$, line leads phase by 30°	$V_L = V_{ph}$
Currents	$I_L = I_{ph}$	$I_L = \sqrt{3} I_{ph}$, line lags phase by 30°
Has a neutral	yes	no
Sums that vanish: $\bar{V}_a + \bar{V}_b + \bar{V}_c = 0$, $\bar{V}_{ab} + \bar{V}_{bc} + \bar{V}_{ca} = 0$, $\bar{I}_a + \bar{I}_b + \bar{I}_c = 0$		

Y- Δ conversion. In general (Tutorial 2), with Z_a, Z_b, Z_c the wye arms at nodes a, b, c and Z_{ab}, Z_{bc}, Z_{ca} the delta sides:

KEY RELATION

$$Z_{ab} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_c}, \quad Z_a = \frac{Z_{ab} Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}}.$$

A delta side is the sum of the pairwise products over the *opposite* wye arm; a wye arm is the product of the two *adjacent* delta sides over their sum. Balanced, both collapse to $Z_\Delta = 3 Z_Y$. Source and load must both be wye to solve a per-phase equivalent; a delta at either end is converted first.

WORKED EXAMPLE · MODULE 2, EXAMPLE 4 (P.60) · CORE · WORKED SOLUTION AGREES WITH THE SOLVED DECK, P.61

In a balanced three-phase circuit, $\bar{V}_{ab} = 173.2\angle 0^\circ$ V. Determine all the phase and line voltages and currents in a Y-connected load having $Z_L = 10\angle 20^\circ \Omega$.

Divide by $\sqrt{3}$ and rotate back by 30° :

$$\bar{V}_{an} = \frac{173.2}{\sqrt{3}} \angle (0^\circ - 30^\circ) = 100 \angle -30^\circ \text{ V},$$

$$\bar{V}_{bn} = 100 \angle -150^\circ, \quad \bar{V}_{cn} = 100 \angle 90^\circ \text{ V},$$

$$\bar{V}_{ab} = 173.2 \angle 0^\circ, \quad \bar{V}_{bc} = 173.2 \angle -120^\circ, \quad \bar{V}_{ca} = 173.2 \angle 120^\circ \text{ V}.$$

In a wye the line current is the phase current:

$$\bar{I}_a = \frac{100 \angle -30^\circ}{10 \angle 20^\circ} = 10 \angle -50^\circ \text{ A}, \quad \bar{I}_b = 10 \angle -170^\circ, \quad \bar{I}_c = 10 \angle 70^\circ \text{ A}.$$

The load is balanced, so $\bar{I}_n = 0$.

Three-phase power, and measuring it

KEY RELATION

$$P_{total} = 3 V_{ph} I_{ph} \cos \theta = \sqrt{3} V_L I_L \cos \theta = \frac{V_L^2}{Z_Y} \cos \theta,$$

$$Q_{total} = 3 V_{ph} I_{ph} \sin \theta = \sqrt{3} V_L I_L \sin \theta = \frac{V_L^2}{Z_Y} \sin \theta.$$

Star or delta alike; θ is the **phase** impedance angle, and Z_Y is the per-phase wye impedance (its magnitude). For a delta load, work per phase with the phase quantities, $V_{ph} = V_L$ and $I_{ph} = I_L / \sqrt{3}$:

$$P_\Delta = 3 I_{ph}^2 R_\Delta = \frac{3 V_L^2}{|Z_\Delta|} \cos \theta, \quad Q_\Delta = 3 I_{ph}^2 X_\Delta, \quad I_{ph} = \frac{V_L}{|Z_\Delta|}.$$

Constant power. The three phases' instantaneous powers each pulse at 2ω , but the pulses are 120° apart and cancel, so the total

$$p(t) = p_a(t) + p_b(t) + p_c(t) = 3 V_{ph} I_{ph} \cos \theta$$

is constant (RMS values) – why a three-phase machine runs with a steady torque where a single-phase one shakes at twice line frequency.

The trap: $S_{total} = 3 \bar{V}_{ph} \bar{I}_{ph}^* \neq \sqrt{3} \bar{V}_L \bar{I}_L^*$ – the magnitudes agree but the angles do not, so build S_{total} from phase quantities.

A wattmeter reads $VI \cos \phi$: voltage coil across the terminals, current coil in series in the branch. Three-wattmeter (wye load, needs a neutral): total $P_A + P_B + P_C$. Two-wattmeter (delta load, no neutral): total $P_1 + P_2$.

WORKED EXAMPLE · MODULE 2, EXAMPLE 7 (P.66) · CORE · CROSS-CHECKED TWO WAYS IN THE DECK

A wye source with a 440 V line voltage supplies load 1, 4 Ω resistors in wye, and load 2, 6 Ω resistors in delta. Find each load's phase voltage and line current, the source line current, and the power delivered.

Voltages. $V_{ph,1} = 440/\sqrt{3} = 254.0$ V; $V_{ph,2} = V_L = 440$ V (delta).

Currents. $I_{ph,1} = 254.0/4 = 63.51$ A = $I_{L,1}$; $I_{ph,2} = 440/6 = 73.33$ A, $I_{L,2} = \sqrt{3} \times 73.33 = 127.0$ A.

Source line current. Both loads resistive, so their line currents are in phase with $\bar{V}_{an}$ and add directly: $I_L = 63.51 + 127.0 = 190.5$ A. *Check* via the per-phase equivalent: $Z_Y = Z_\Delta/3 = 2 \Omega$ in parallel with the 4 Ω wye load gives $4||2 = 1.333 \Omega$, so $254.0/1.333 = 190.5$ A.

Power. Resistive loads, $\cos \theta = 1$: $P = \sqrt{3} V_L I_L \cos \theta = \sqrt{3} \times 440 \times 190.5 = 145.2$ kW. *Check per load:* load 1 dissipates $3 \times 254.0^2/4 = 48.4$ kW, load 2 dissipates $3 \times 440^2/6 = 96.8$ kW, summing to 145.2 kW.

Single-phase R, L and C

Every branch sees the same source voltage E , so currents add as phasors:

KEY RELATION

$$\bar{I}_R = \frac{E}{R} \angle 0^\circ, \quad \bar{I}_L = \frac{E}{X_L} \angle -90^\circ, \quad \bar{I}_C = \frac{E}{X_C} \angle +90^\circ, \quad X_L = 2\pi f L, \quad X_C = \frac{1}{2\pi f C},$$

$$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C = I_R - j(I_L - I_C), \quad P = E I_R, \quad Q = E(I_L - I_C), \quad PF = \frac{I_R}{|\bar{I}|}.$$

WORKED EXAMPLE · 1-PH LAB MANUAL, TABLE 1.6 STATION 1 · CORE ·

Station 1: $R = 171.43 \Omega$, $L = 0.8$ H, $C = 2.2 \mu\text{F}$, $E = 120$ V at 60 Hz, ideal elements. $X_L = 2\pi(60)(0.8) = 301.6 \Omega$ and $X_C = 1/(2\pi(60)(2.2 \times 10^{-6})) = 1205.7 \Omega$, so $I_R = 0.700$ A, $I_L = 0.398$ A and $I_C = 0.0995$ A.

STATION 1, THE BASE CASES AND RLC (IDEAL)

ON	A/M5	P (METER)	Q	S	PF
R	0.700 A	84.0 W (P1)	0	84.0 VA	1.000
L	0.398 A	0 (B/M11)	+47.7 VAR	47.7 VA	0 (lags 90°)
C	0.100 A	0 (B/M11)	-11.9 VAR	11.9 VA	0 (leads 90°)
RLC	0.761 A	84.0 W (P1)	+35.8 VAR	91.3 VA	0.920 lag

Magnetic circuits

Module 3: every machine in Modules 4–7 is the same object – a magnetic circuit with windings on it.

Fields, MMF and reluctance

A coil of N turns carrying I drives flux around a core exactly as an EMF drives current through a resistive network:

KEY RELATION

$$B = \frac{\phi}{A} [\text{T}], \quad H = \frac{NI}{\ell} [\text{A}\cdot\text{t/m}], \quad \mathcal{F} = NI = H\ell = \mathcal{R}\phi,$$

$$\mathcal{R} = \frac{\ell}{\mu A} \left[\frac{\text{A}\cdot\text{t}}{\text{Wb}} \right], \quad \mu = \mu_r \mu_0, \quad \mu_0 = 4\pi \times 10^{-7} \text{ H/m}.$$

ℓ is the *mean* path, along the centre line of the core, and A the cross-section. One loop carries the same flux in every segment, so B is largest where the core is thinnest. Reluctance is constant in air ($\mu_r = 1$) and not constant in steel.

THE ANALOGY

MAGNETIC	ELECTRIC
MMF $\mathcal{F} = NI$	EMF e
flux ϕ	current i
reluctance $\ell/(\mu A)$	resistance $\ell/(\sigma A)$
$B = \phi/A, H = \mathcal{F}/\ell$	$J = i/A, E = e/\ell$

Series reluctances add and parallel ones add as reciprocals. Round any loop the MMF drops equal the sources (magnetic KVL, $\sum NI = \sum \mathcal{R}\phi$); the fluxes into a junction sum to zero (magnetic KCL). Air's permeability is about a thousand times smaller than iron's, so a millimetre

of gap can out-reluctance the whole iron path round it: in a machine almost all the MMF is spent crossing the gap.

B-H, hysteresis and core loss

$B = \mu H$ bends because the domains align until the core saturates: a linear region, a knee, then saturation, where $\mu = B/H$ collapses toward air's value, $\mathcal{R}$ grows, and doubling the MMF buys far less than double the flux. Machines run **near the knee**. Under AC the core traces a hysteresis loop: residual B_r at $H = 0$, coercive H_c to bring B back to zero. The loop's **area** is energy lost per cycle per unit volume. Soft materials (narrow loop) make cores; hard ones (wide loop, large B_r) make permanent magnets. A smaller amplitude traces a nested minor loop.

A changing flux also induces EMF inside the iron, which drives circulating eddy currents. Thin laminations (0.35–0.5 mm, varnish-insulated) along the flux direction break those loops.

KEY RELATION

$$P_h = k_h V B_{max}^n f \quad (n = 1.5-2.5), \quad P_e = k_e V (f t B_{max})^2, \quad P_{core} = P_h + P_e.$$

V is the core volume and t the lamination thickness. Doubling f doubles P_h and quadruples P_e ; thinner laminations cut only P_e .

The analogy breaks in two more places. **Leakage**: some flux leaves the core and returns through air without linking the far winding, so more MMF is needed – and why a real transformer has a leakage reactance. **Fringing**: flux at a gap spreads sideways, so the gap's effective area exceeds the pole face,

$$A_g = (w + \ell_g)(d + \ell_g) \quad \text{or a stated percentage,}$$

and $B_g < B_{core}$. Apply it to the gap only.

Solving a core

Split the core into segments, take $\mathcal{R} = \ell/(\mu A)$ for each, combine them as a network, then $\phi = \mathcal{F}/\mathcal{R}$ and $B = \phi/A$. A gap shortens its leg's iron by ℓ_g . A core with two windings: their MMFs add or subtract by the winding senses (right-hand rule). With no μ_r given, work from the B - H curve instead: $H = NI/\ell$, read B , then $\phi = BA$; going the other way, $\phi \rightarrow B \rightarrow H$ from the curve, then $NI = \sum H\ell$.

WORKED EXAMPLE · MODULE 3, EXAMPLE 1 · CORE · PP.28--32

$N = 250$, $\mu_r = 3000$, depth 10 cm, no gap, 2 A. Mean paths 37.5, 35, 37.5 and 35 cm:

$$\mathcal{R} = 9947.184 \times 2 + 9284.038 + 6189.359 = 35\,367.765 \text{ A}\cdot\text{t/Wb},$$

$$\phi = \frac{500}{35\,367.765} = 0.014137 \text{ Wb}, \quad B = 1.414 \text{ T}.$$

Now cut a 0.5 mm gap, area +2.5% for fringing, and hold the same flux: $\mathcal{R}_g = 38\,818.279$, the gapped leg's iron $\mathcal{R}'_2 = 9270.775$, so $\mathcal{R} = 74\,172.781$ and $I = \mathcal{R}\phi/N = 4.195 \text{ A}$ – about 2.1× the current, for half a millimetre of air.

WORKED EXAMPLE · MODULE 3, EXAMPLE 2 · DC MACHINE, TWO GAPS · P.33

Stator 185 cm, 428.36 cm²; rotor 10 cm, 424.12 cm²; two 0.5 mm gaps of 450 cm²; $\mu_r = 2500$; $NI = 600$.

$$\mathcal{R}_s = 13\,747, \quad \mathcal{R}_r = 750.5, \quad \mathcal{R}_g = 8842 \text{ each},$$

$$\mathcal{R} = 13\,747 + 750.5 + 2(8842) = 32\,182 \text{ A}\cdot\text{t/Wb},$$

$$\phi = 600/32\,182 = 18.64 \text{ mWb}, \quad B_g = 0.414 \text{ T}, \quad B_s = 0.435 \text{ T}.$$

The flux crosses the gap *twice*; with one gap you get 25.7 mWb, the usual slip. The published solution (slide 36) adds $\mathcal{R}_r$ twice instead of $\mathcal{R}_g$ twice and gets 24 090 and 0.0249 Wb.

Induced EMF and force

KEY RELATION

$$e = N \frac{d\phi}{dt} = \frac{d\lambda}{dt}, \quad \lambda = N\phi; \quad e = (\vec{v} \times \vec{B}) \cdot \vec{\ell} = v\ell B; \quad \vec{F} = i(\vec{\ell} \times \vec{B}), \quad F = i\ell B.$$

Lenz: the induced current's flux opposes the *change* in flux, which is the minus sign in $e = -N d\phi/dt$ – a statement about direction, not size. A conductor moving through a field generates ($e = v\ell B$); a conductor carrying current in a field is pushed ($F = i\ell B$). Every machine does both at once. Example: 100 turns, $\phi = 0.01 \sin 377t$ gives $e = 377 \cos 377t \text{ V}$.

CHECK YOURSELF

A core is already past the knee of its B - H curve. You double the coil current. What happens to the flux and to the reluctance?

Answer. The flux rises by much less than a factor of two: past the knee μ has collapsed toward air's value, so $\mathcal{R} = \ell/(\mu A)$ has grown, and the extra MMF works against a larger opposition than the first half did.

Transformers

Module 4. The worked solutions on the slides are blank, so every answer in this section is ours, recomputed and checked against the transformers lesson. A lagging current has a *negative* angle.

The ideal transformer

Two coils linked by the same flux on an iron core. AC on the primary makes a time-varying core flux, Faraday induces an EMF in the secondary, and once a load is connected Lenz's law sets the secondary current's direction: its flux opposes the change in the original flux. Ideal means $\mu \rightarrow \infty$ (so $\mathcal{R} = 0$), no leakage, and no core or winding losses. With $\phi = \phi_m \sin \omega t$, $v_1 = N_1 \omega \phi_m \cos \omega t$ and $v_2 = N_2 \omega \phi_m \cos \omega t$, so each winding's RMS voltage is

KEY RELATION

$$V = \frac{\omega \phi_m N}{\sqrt{2}} = 4.44 f \phi_m N, \quad a = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1},$$

$$S_1 = \bar{V}_1 \bar{I}_1^* = (a \bar{V}_2) \left(\frac{\bar{I}_2^*}{a} \right) = \bar{V}_2 \bar{I}_2^* = S_2, \quad Z'_2 = \frac{\bar{V}_1}{\bar{I}_1} = a^2 Z_2.$$

The current ratio comes from Ampère's law around the core, $\mathcal{R}\phi = N_1 i_1 - N_2 i_2$, which is zero when $\mathcal{R} = 0$. The impedance seen through the transformer scales by the square because the voltage scales by a and the current by $1/a$; referring with a single factor of a is the commonest error in this module. **Dots:** if the dotted primary terminal is the higher potential, so is the dotted secondary one; current *into* the primary dot leaves *out of* the secondary dot. Uses: stepping up ($a < 1$) or down ($a > 1$), isolation ($a = 1$), measurement, and impedance matching. Real units run 92% (small) to 99% (large) efficient.

Impedance matching. A source delivers the most power when the load it sees matches its own impedance. Seen through the transformer the load is $a^2 Z_L$, so the matching turns ratio is

KEY RELATION

$$a = \sqrt{|Z_s|/|Z_L|}.$$

WORKED EXAMPLE · MODULE 4, EXAMPLE (P.13) · CORE · OURS

A 240/120 V, 60 Hz ideal transformer rated 5 kVA. (a) The turns ratio. (b) The rated currents. (c) The currents when it delivers 3.2 kW at 0.8 lagging and rated secondary voltage.

Answer. (a) $a = 240/120 = 2$. (b) $5000/240 = 20.83$ A (primary) and $5000/120 = 41.67$ A (secondary). (c) $S = 3.2/0.8 = 4$ kVA, so with $\bar{V}_2$ as reference $\bar{I}_2 = 33.33\angle -36.87^\circ$ A and $\bar{I}_1 = \bar{I}_2/a = 16.67\angle -36.87^\circ$ A.

The real transformer and its equivalent circuit

A practical unit departs from the ideal in four ways: the windings have resistance, some flux leaks (links one winding only), the core's permeability is finite, and the core has real and reactive losses. A coil's inductance follows from Module 3, $\lambda = N\phi = N^2i/\mathcal{R}$, so $L = N^2/\mathcal{R}$. The primary flux is the mutual flux plus a leakage flux, $\phi_1 = \phi_m + \phi_{\ell 1}$, and the winding equation splits the same way:

KEY RELATION

$$v_1 = r_1 i_1 + L_1 \frac{di_1}{dt} + \underbrace{N_1 \frac{d\phi_m}{dt}}_{e_1}, \quad v_2 = r_2 i_2 + L_2 \frac{di_2}{dt} + e_2,$$

with $\lambda_{\ell 1} = N_1 \phi_{\ell 1} = L_1 i_1$, and likewise on the secondary. So each winding gets a series resistance (R_1, R_2 : $I^2 R$ copper loss) and a leakage reactance ($X_1 = \omega L_1, X_2 = \omega L_2$).

The exciting branch. With finite reluctance, $\mathcal{R}_c \phi_m = N_1 i_1 - N_2 i_2$ is no longer zero: the primary draws an extra magnetizing current $i_m = i_1 - (N_2/N_1)i_2$ to set up the flux, and $e_1 = N_1 d\phi_m/dt = L_m di_m/dt$ with $L_m = N_1^2/\mathcal{R}_c$. Hysteresis and eddy-current loss add a current in phase with E_1 . The two become a shunt branch $R_c \parallel jX_m$ across E_1 :

KEY RELATION

$$\bar{I}_e = \bar{I}_c + \bar{I}_m = (G_c - jB_m) \bar{E}_1, \quad G_c = \frac{1}{R_c}, \quad B_m = \frac{1}{X_m}.$$

$\bar{I}_e$ is the exciting current, $\bar{I}_c$ the core-loss current and $\bar{I}_m$ the magnetizing current. R_c and X_m are very large next to the winding resistances and leakage reactances. An ideal transformer sits between E_1 and E_2 .

Referral. Move the secondary across the ideal transformer to the primary side and the ideal transformer disappears:

KEY RELATION

$$R'_2 = a^2 R_2, \quad X'_2 = a^2 X_2, \quad Z'_L = a^2 Z_L, \quad \bar{V}'_2 = a \bar{V}_2, \quad \bar{I}'_2 = \bar{I}_2/a.$$

To the secondary side it runs the other way: $R_1/a^2, X_1/a^2, R_c/a^2, X_m/a^2$, with $\bar{V}_1/a$ and $a\bar{I}_1$. Then solve it as an ordinary circuit, in phasors.

Approximate circuits. The exciting current is 3–5% of rated, and the series drops are small, so $V_1 \approx E_1$. Moving the shunt branch to the terminals lets the series elements combine:

KEY RELATION

$$R_{e1} = R_1 + a^2 R_2, \quad X_{e1} = X_1 + a^2 X_2, \quad \bar{V}_1 = a\bar{V}_2 + \bar{I}'_2 (R_{e1} + jX_{e1}).$$

Referred to the secondary, $R_{e2} = R_{e1}/a^2$ and $X_{e2} = X_{e1}/a^2$. That is the 1st approximation; the 2nd drops the shunt branch altogether, which is valid in power-system studies unless efficiency or the exciting current itself is the question.

WORKED EXAMPLE · MODULE 4, EXAMPLE (P.24) · CORE · OURS

25 kVA, 440/220 V, 60 Hz; $R_1 = 0.16$, $R_2 = 0.04$, $X_1 = 0.32$, $X_2 = 0.08$, $R_c = 270$, $X_m = 100 \Omega$ (shunt on the HV side as drawn). It delivers 20 kW at 0.8 lagging at rated voltage to a load on the LV side. Find the primary terminal voltage.

$a = 2$, so $R'_2 = 0.16$, $X'_2 = 0.32 \Omega$. Load current $\bar{I}_2 = 20000/(0.8 \times 220) = 113.64 \angle -36.87^\circ$ A, referred $\bar{I}'_2 = 56.82 \angle -36.87^\circ$ A, with $\bar{V}'_2 = 440 \angle 0^\circ$ V.

$$\begin{aligned} \bar{E}_1 &= \bar{V}'_2 + \bar{I}'_2(0.16 + j0.32) = 458.27 \angle 1.14^\circ \text{ V}, \\ \bar{I}_c &= \frac{\bar{E}_1}{270} = 1.697 \angle 1.14^\circ, \quad \bar{I}_m = \frac{\bar{E}_1}{j100} = 4.583 \angle -88.86^\circ \text{ A}, \\ \bar{I}_e &= \bar{I}_c + \bar{I}_m = 4.887 \angle -68.54^\circ \text{ A}, \quad \bar{I}_1 = \bar{I}'_2 + \bar{I}_e = 61.03 \angle -39.28^\circ \text{ A}, \\ \bar{V}_1 &= \bar{E}_1 + \bar{I}_1(0.16 + j0.32) = 478.4 \angle 2.16^\circ \text{ V}. \end{aligned}$$

The approximate circuit, $\bar{V}_1 = 440 + \bar{I}'_2(0.32 + j0.64)$, gives $476.7 \angle 2.19^\circ$ V, 0.36% below the exact answer.

Finding the parameters: two tests

Open-circuit test – rated voltage on the LV side, HV open; measure P_{oc} , V_{oc} , I_{oc} on the LV side. The series branch carries only the tiny exciting current, so all of P_{oc} is core loss and the measurement sees only the shunt branch. The LV side is chosen because a rated-voltage source is easier there, the meters are cheaper, and the small current reads more accurately. The results are on the LV side; multiply by a^2 to refer them to the HV side.

KEY RELATION

$$R_{c2} = \frac{V_{oc}^2}{P_{oc}}, \quad Y_{o2} = \frac{1}{R_{c2}} - j \frac{1}{X_{m2}}, \quad |Y_{o2}| = \frac{I_{oc}}{V_{oc}}, \quad X_{m2} = \frac{1}{\sqrt{|Y_{o2}|^2 - 1/R_{c2}^2}}.$$

Short-circuit test – LV shorted, HV on a low variable voltage raised until rated current flows; measure P_{sc} , V_{sc} , I_{sc} on the HV side. The HV side is chosen because its rated current is the

smaller one, so the wattmeter and ammeter can have lower current ratings. The voltage is so low that the core flux and core loss are negligible, so all of P_{sc} is the rated copper loss, and the shunt branch drops out.

KEY RELATION

$$R_{e1} = \frac{P_{sc}}{I_{sc}^2}, \quad |Z_{e1}| = \frac{V_{sc}}{I_{sc}}, \quad X_{e1} = \sqrt{|Z_{e1}|^2 - R_{e1}^2},$$

$$R_1 = a^2 R_2 = \frac{1}{2} R_{e1}, \quad X_1 = a^2 X_2 = \frac{1}{2} X_{e1}$$

when the design splits the losses and leakage equally. Slide 35 puts the short-circuit voltage at “20–30% of rated”, but its own example (p.40) uses 48 V on a 2400 V winding, which is 2%, and Problem Set 4’s P5 uses 5.5 kV on 66 kV, 8.3%. A few percent is typical; read the slide’s figure as a slip.

Regulation and efficiency

KEY RELATION

$$\varepsilon = \frac{|V_{2,nl}| - |V_{2,fl}|}{|V_{2,fl}|} \times 100\% = \frac{|V_1|/a - |V_2|}{|V_2|} \times 100\%,$$

with the magnetizing branch usually neglected. It measures the drop across the transformer’s internal impedance, so an ideal transformer has $\varepsilon = 0$, and a leading load can make it negative. Copper loss grows with the square of load current, while core loss is set by the flux and stays put from no load to full load. The open-circuit test measures the core loss and the short-circuit test the full-load copper loss, so $P_{core} = P_{oc}$ and $P_{cu,fl} = P_{sc,fl}$. At a fraction x of full load, $I_x^2 R_{e1} = x^2 I_{fl}^2 R_{e1}$ and $P_{out,x} = x V_2 I_{fl} \cos \theta_2$:

KEY RELATION

$$\eta_x = \frac{P_{out}}{P_{out} + P_{core} + P_{cu}} = \frac{x S \cos \theta}{x S \cos \theta + P_{oc} + x^2 P_{sc,fl}} = 1 - \frac{x^2 P_{sc,fl} + P_{oc}}{x P_{out,fl} + x^2 P_{sc,fl} + P_{oc}}.$$

Beyond the peak, setting $d\eta/dx = 0$ puts the peak where the two losses are equal: $x^2 P_{sc,fl} = P_{oc}$, so $x = \sqrt{P_{oc}/P_{sc,fl}}$.

WORKED EXAMPLE · MODULE 4, EXAMPLE (P.40) · CORE · OURS

50 kVA, 2400/240 V, 60 Hz. Open circuit on the LV side: 240 V, 5.4 A, 186 W. Short circuit on the HV side: 48 V, 20.8 A, 620 W. (a) The approximate circuit referred to the primary. (b) Regulation and efficiency at rated load, 0.8 lagging, rated secondary voltage.

Shunt branch (LV side). $R_{c2} = 240^2/186 = 309.7 \Omega$; $|Y| = 5.4/240 = 0.0225 \text{ S}$, $G = 0.003229 \text{ S}$, $B = 0.02227 \text{ S}$, so $X_{m2} = 44.91 \Omega$. Referred to the HV side, multiply by $a^2 = 100$: $R_{c1} = 30.97 \text{ k}\Omega$, $X_{m1} = 4.49 \text{ k}\Omega$.

Series branch (HV side). $R_{e1} = 620/20.8^2 = 1.433 \Omega$, $|Z_{e1}| = 48/20.8 = 2.308 \Omega$,
 $X_{e1} = 1.809 \Omega$.

Regulation. $I_1 = 50000/2400 = 20.83 \angle -36.87^\circ \text{ A}$, so $\bar{V}_1 = 2400 + \bar{I}_1(1.433 + j1.809) = 2446.5 \angle 0.29^\circ \text{ V}$ and $\varepsilon = 46.5/2400 = 1.94\%$.

Efficiency. $P_{out} = 40 \text{ kW}$, $P_{cu} = 20.83^2 \times 1.433 = 622 \text{ W}$ at rated current (the test's 620 W was at 20.8 A), $P_{core} = 186 \text{ W}$: $\eta = 40000/40808 = 98.02\%$. Peak efficiency (beyond the deck) at $x = \sqrt{186/622} = 0.547$, i.e. 54.7% load, where $\eta = 98.33\%$ at 0.8 pf.

Three-phase transformers (not in the deck)

The outline lists them under Module 4, but the slides stop at single-phase units, so this is ours. Three single-phase windings, as a bank of three units or on one core, each side connected in Y or Δ . Solve one phase with the single-phase circuit, then convert to line quantities as in the three-phase sections: each winding sees a phase voltage, so a Y- Δ bank of winding ratio a has a line-voltage ratio of $\sqrt{3}a$ and a 30° shift, and each unit of a bank carries a third of the total kVA.

CHECK YOURSELF

A transformer has $a = 10$. A 5Ω load sits on the secondary. What impedance does the primary source see?

Answer. $Z_1 = a^2 Z_2 = 100 \times 5 = 500 \Omega$, not 50Ω .

DC machines

Module 5.

Field winding on the stator, armature on the rotor – the opposite of every AC machine in the next two sections. A commutator mechanically rectifies: the armature carries AC, the terminals see DC. Two equations, sharing one constant K set by the winding geometry, carry the module:

KEY RELATION

$$E_a = K\phi\omega, \quad T = K\phi I_a, \quad E_a I_a = T\omega.$$

$$\text{motor: } V_t = E_a + I_a R_a \quad \text{generator: } V_t = E_a - I_a R_a.$$

At standstill $E_a = 0$, so $I_a = V_t/R_a$ is many times rated – hence starting resistance, needed only until E_a grows enough to limit the current itself. Since

KEY RELATION

$$\omega = \frac{V_t - I_a R_a}{K\phi},$$

weakening the field *raises* speed, because ϕ sits in the denominator. Field-to-armature connection sets the machine type: separately excited (ϕ fixed by an independent supply), shunt (field across the armature terminals, speed droops only slightly with load), series (field in series with the armature, so flux rises with load current and torque goes as I_a^2 – large low-speed torque, but an unloaded series motor has almost no flux and can run away), and compound (both fields, between the two).

CHECK YOURSELF

A shunt DC motor runs at steady speed on a fixed terminal voltage. You reduce the field current. What happens to the speed?

Answer. The speed rises: with V_t fixed, $\omega = (V_t - I_a R_a)/(K\phi)$, and ϕ is in the denominator.

CHECK YOURSELF

A DC motor with $R_a = 0.5 \Omega$ is connected directly to a 250 V supply at standstill. Estimate the armature current at the instant of connection.

Answer. At standstill $E_a = 0$, so $I_a = V_t/R_a = 250/0.5 = 500$ A, many times rated. As the machine accelerates E_a grows in proportion to speed and the current falls to $(V_t - E_a)/R_a$.

Synchronous machines

Module 6.

DC-excited field winding on the rotor, three-phase armature on the stator – the reverse of a DC machine, no commutator needed. The rotating stator field turns only at synchronous speed;

the rotor must match it exactly or average torque is zero, so the machine produces no starting torque on its own:

KEY RELATION

$$n_s = \frac{120f}{p} [\text{rpm}], \quad \bar{E}_f = \bar{V}_t + jX_s \bar{I}_a \text{ (generator)}, \quad P = \frac{3E_f V_t}{X_s} \sin \delta.$$

δ is not free: it settles wherever the electrical power out must match the mechanical power in. Power rises with δ only to $\delta = 90^\circ$ (the maximum deliverable power is $3E_f V_t / X_s$, the steady-state stability limit); past it the electrical power the machine can deliver falls while the mechanical input keeps rising, so the rotor accelerates away and loses synchronism.

On a stiff grid (V_t, f fixed, real power set by the prime mover), the field current instead controls reactive power: over-excite and the machine supplies Q with a lagging armature current; under-excite and it absorbs Q with a leading current. **Watch the current direction:** earlier sections read Q from a load, where a lagging current means the load absorbs Q ; here $\bar{I}_a$ is defined out of the machine, so the same lagging current means the machine *delivers* Q – same physics, opposite bookkeeping. A synchronous motor deliberately over-excited to supply Q to a plant is a synchronous condenser: power-factor correction done with a machine instead of a capacitor.

WORKED EXAMPLE · PER-UNIT EXCITATION

Take $V_t = 1\angle 0^\circ$ pu, $X_s = 1$ pu, $P = 0.5$ pu and an over-excited $E_f = 1.5$ pu. Then $\delta = 19.5^\circ$ and

$$\bar{I}_a = \frac{\bar{E}_f - \bar{V}_t}{jX_s} = 0.649\angle -39.6^\circ \text{ pu},$$

a lagging current, and $\bar{S} = \bar{V}_t \bar{I}_a^* = 0.5 + j0.41$ pu: reactive power flows out of the machine. Drop E_f to 1.05 pu and $\bar{I}_a$ swings to $+8.7^\circ$, leading, with $Q = -0.08$ pu – absorbed.

Induction motors

Module 7.

If the rotor turned at synchronous speed it would see a stationary field, no flux change, no induced current, no torque – it must run slower, by the slip:

KEY RELATION

$$s = \frac{n_s - n}{n_s}, \quad \frac{R_2}{s} = R_2 + R_2 \frac{1-s}{s}.$$

The first piece is genuine rotor resistance, dissipating rotor copper loss; the second is not a resistance at all, and the power it absorbs is where mechanical power leaves the circuit. At

$s = 1$ it vanishes (a stalled motor delivers no mechanical power); as $s \rightarrow 0$ it grows without bound (almost all air-gap power is converted, not dissipated). Torque as a function of slip:

KEY RELATION

$$T = \frac{3}{\omega_s} \cdot \frac{V_1^2(R_2/s)}{(R_1 + R_2/s)^2 + (X_1 + X_2)^2}, \quad s_{max} = \frac{R_2}{\sqrt{R_1^2 + (X_1 + X_2)^2}}.$$

R_2 cancels when s_{max} is substituted back: rotor resistance moves *where* breakdown torque sits, not *how high* – the basis of wound-rotor starting. Torque goes as V_1^2 at every slip, so a lower supply voltage scales the whole curve down without moving the breakdown slip – reduced-voltage starting is gentle on the supply and weak on torque. A third handle, supply frequency, moves n_s itself: what a variable-frequency drive does. Parameter tests mirror the transformer's: no-load (rotor branch effectively open) gives the magnetizing branch and rotational losses; blocked-rotor (reduced voltage, $s = 1$) gives the series resistances and leakage reactances.

CHECK YOURSELF

A 4-pole 60 Hz induction motor runs at 1728 rpm. Compute the slip and the fraction of air-gap power lost as rotor copper loss.

Answer. $n_s = 120 \times 60/4 = 1800$ rpm, $s = (1800 - 1728)/1800 = 0.04$: 4% of the air-gap power is rotor copper loss, the other 96% becomes mechanical power.

Practice problems

From the filed decks, the problem sets and the lessons' check-yourself questions, same numbers. Problem Set 4's solutions are not posted yet, so its problems are stated here without answers.

1. (**Module 2, Example 1, p.22**) A load on a 79.7 kV busbar is $Z = 80 + j60 \Omega$. Find the real and reactive power it absorbs and its power factor.

Answer: $\bar{I} = 797 \angle -36.87^\circ$ A, $S = 63.52 \angle 36.87^\circ$ MVA, so $P = 50.82$ MW, $Q = 38.11$ MVAR, $PF = 0.8$ **lagging**.

2. A balanced delta load of $30 \angle 40^\circ \Omega$ per phase sits on a 208 V line. What impedance goes into the per-phase equivalent circuit, and what voltage does that circuit see?

Answer: $Z_Y = Z_\Delta/3 = 10 \angle 40^\circ \Omega$, the angle unchanged. Drawn line-to-neutral, it sees $208/\sqrt{3} = 120.1$ V.

3. (PS2 P2) A balanced delta load of $12 + j9 \Omega$ per phase on a 208 V, 60 Hz source. Find the line current, P , Q and the power factor.
Answer (official): $I_{ph} = 208/15 = 13.87 \text{ A}$, $\bar{I}_a = 24.02 \angle -66.87^\circ \text{ A}$ ($\bar{V}_{ab}$ reference), $P = 6923 \text{ W}$, $Q = 5192 \text{ VAR}$, $PF = 0.8$ lagging.
4. (PS2 P5) An 18Ω delta in parallel with a 12Ω wye, both resistive, fed from 346.41 V line-to-line through $j3 \Omega$ per phase. Find the source current and the power it supplies.
Answer (official): $18 \Omega \Delta \rightarrow 6 \Omega \text{ Y}$, $6 \parallel 12 = 4 \Omega$; per phase $200/(4 + j3)$, so $\bar{I}_s = 40 \angle -36.87^\circ \text{ A}$, $P = 19.2 \text{ kW}$ (all in the loads) and $Q = 14.4 \text{ kVAR}$ (all in the line).
5. (PS2 P7) 15 kW at 0.6 lagging in parallel with 10 kVA at 0.8 leading, on 480 V. Find the combined power factor, the line current, and the delta capacitor bank that makes it unity.
Answer (official): $S = (15 + j20) + (8 - j6) = 26.93 \angle 31.33^\circ \text{ kVA}$, $PF = 0.854$ lagging, $I_L = 32.4 \text{ A}$. Cancelling 14 kVAR across three delta legs at 480 V needs $X_C = 480^2/4667 = 49.37 \Omega$ per leg, and the line current falls to 27.66 A.
6. (PS3 P1) $\mu_r = 800$, 500 turns, $\phi = 5 \text{ mWb}$. *Answer (official, as the lesson's):* $\mathcal{R} = 59\,683 + 72\,946 + 119\,366 = 251\,995 \text{ A}\cdot\text{t/Wb}$; $I = 2.52 \text{ A}$; $B_{top} = 0.667 \text{ T}$, $B_{right} = 2.0 \text{ T}$.
7. (PS3 P2) Three legs, 300 turns at 1 A on the centre, $\mu_r = 1500$, gaps $+5\%$. The text gives the left gap 0.050 cm and the right 0.070 cm, but the official solution follows the figure: left 0.07 cm, right 0.05 cm. *Answer (official):* $\mathcal{R}_L = 319\,720$, $\mathcal{R}_R = 276\,442 \text{ A}\cdot\text{t/Wb}$; $\phi_c = 1.468 \text{ mWb}$, $\phi_L = 0.681$, $\phi_R = 0.787 \text{ mWb}$; $B_{gL} = 0.185 \text{ T}$, $B_{gR} = 0.214 \text{ T}$ (printed 0.215, from the rounded 0.79 mWb). The smaller gap carries more flux.
8. (PS3 P3) *Answer (official):* $\mathcal{R} = 76\,630$; aiding $\phi = 6.52 \text{ mWb}$ (the official reading), opposing 1.30 mWb. Say which you read from the winding senses.
9. (PS3 P5) 2 A upward along 1 m of wire in 0.5 T pointing right. *Answer (official):* $F = i\ell B = 1 \text{ N}$, into the page.
10. Mean path 60 cm, 16 cm^2 , $\mu_r = 2000$, a 1 mm gap at $+4\%$, 400 turns, 1.0 T in the core. Find the current, the gap flux density, and the current without the gap.
Answer: $\mathcal{R}_m = 148\,959$, $\mathcal{R}_g = 478\,230$, $I = 2.51 \text{ A}$, $B_g = 0.962 \text{ T}$; without the gap 0.60 A.
11. A transformer has $a = 10$. A 5Ω load sits on the secondary. Predict the impedance the primary source sees.
Answer: $Z_1 = a^2 Z_2 = 100 \times 5 = 500 \Omega$, not 50Ω .
12. (PS4 P1) $v_{in} = 18 \sin 10t \text{ V}$ behind 18Ω , a 36Ω shunt, an ideal 3:1 transformer, 2Ω in series, then an ideal 1:2 transformer feeding 8Ω . Find $v_{out}(t)$ across the 8Ω . The second transformer's secondary dot is at the bottom: mind the sign. *Solutions not posted yet.*
13. (PS4 P2) $v = 10\sqrt{2} \sin 2t \text{ V}$ with 1800Ω internal resistance, coupled through an ideal transformer to a 50Ω load. Find the turns ratio a for maximum power transfer and the average power the load then receives. *Solutions not posted yet.*

14. (PS4 P3) $v_s = 282.8 \sin 377t$ V and $i_s = 7.07 \sin(377t - 36.87^\circ)$ A on the secondary, $N_1:N_2 = 100:200$ ($a = 0.5$); referred to the primary $R_{eq} = 0.2$, $X_{eq} = 0.75$, $R_C = 300$, $X_M = 80 \Omega$. On the first approximate circuit find $i_1(t)$, the regulation and the efficiency. *Solutions not posted yet.*
15. (PS4 P4) 2000:500 turns, $R_1 = 2$, $R_2 = 0.125$, $X_1 = 8$, $X_2 = 0.5 \Omega$, a 12Ω resistive load, 1000 V on the primary. Neglecting the exciting current, find V_2 and the regulation. *Solutions not posted yet.*
16. (PS4 P5) 15 MVA, 66/11.5 kV step-down. Short circuit (LV shorted): rated current at 5.5 kV, 100 kW on the HV side. Open circuit (HV open): 11.5 kV, 30 A, 65 kW on the LV side. Find R_{eq1} , X_{eq1} , G_c and B_m referred to the HV side, and the efficiency at 12 MW, 0.8 lagging, rated voltage. *Solutions not posted yet.*
17. (PS4 P6) 50 kVA, 2400/240 V, 60 Hz step-down at the end of a 2400 V feeder of $1.0 + j2.0 \Omega$; the transformer's series impedance is $1.0 + j2.5 \Omega$, both referred to the HV side. At rated load, 0.8 lagging and rated secondary voltage, exciting current neglected, find the primary terminal voltage, the sending-end voltage, and the sending-end P and Q . *Solutions not posted yet.*
18. A shunt DC motor runs at steady speed on a fixed terminal voltage. You reduce the field current. Predict what happens to the speed, and name the term that drives it.
Answer: The speed rises: with V_t fixed, $\omega = (V_t - I_a R_a)/(K\phi)$, and ϕ is in the denominator.
19. A synchronous generator on a stiff grid delivers 5 MW. You raise its field current and leave the prime mover alone. Predict what happens to the real and reactive power.
Answer: Real power stays at 5 MW – the prime mover sets it and the grid holds V_t and f . Raising E_f makes the machine supply more reactive power, and δ falls to keep P constant.
20. A 4-pole 60 Hz induction motor runs at 1728 rpm. Compute the slip and the fraction of air-gap power lost as rotor copper loss.
Answer: $n_s = 120 \times 60/4 = 1800$ rpm, $s = (1800 - 1728)/1800 = 0.04$, so 4% of the air-gap power is rotor copper loss.
21. Two identical wound-rotor induction motors differ only in added rotor resistance. Compare their starting torque and their breakdown torque.
Answer: The one with added resistance has the higher starting torque, because $s_{max} = R_2/\sqrt{R_1^2 + X^2}$ moves toward $s = 1$. Their breakdown torques are identical: R_2 cancels out of the torque expression once s_{max} is substituted back in.

Lesson sources: lessons/ECE260/claude_lessons/ece260-ac-circuits/src/ece260_ac_circuits.jsx and lessons/ECE260/claude_lessons/ece260-course-overview/src/ece260_course_overview.jsx, read 2026-09-27; ECE 260 Module 1 and 2 slides and the single-phase lab manual (S. Pirooz Azad, Fall 2026), filed under lessons/ECE260/materials/. Extended 2026-10-05 from lessons/ECE260/claude_lessons/ece260-magnetic-circuits/src/ece260_magnetic_circuits.jsx and its companion, the Module 3 and 4

slides, Tutorial 2, and the Problem Set 2 solutions; Problem Set 3 answers checked against its official solutions (filed 2026-10-07). Transformers rechecked 2026-10-07 against lessons/ECE260/claude_lessons/ece260-transformers/src/ece260_transformers.jsx; Problem Set 4 (filed 2026-10-07) is stated without answers until its solutions post. The Module 4 examples have no published solutions, so their answers here are ours. The course bans generative AI on assessments: these notes are study material, never an answer to graded work.