

Magnetic circuits

Key-points companion for <https://lessons.ihsan.cc/ece260/ece260-magnetic-circuits/> – formulas, the worked examples in full, and practice with answers.

BEFORE QUIZ 2

Quiz 2 (magnetic circuits, group quiz) is Wednesday 7 October at 16:30. The Module 3 videos are filed beside the slides and are worth watching first; this sheet is built from the 42 slides and Problem Set 3, not from the videos. Its Problem Set 3 answers are checked against the official solutions.

Fields, MMF and reluctance

ON THE FORMULA SHEET

$$B = \frac{\phi}{A} \text{ [T]}, \quad H = \frac{NI}{\ell} \text{ [A}\cdot\text{t/m]}, \quad \mathcal{F} = NI = H\ell = \mathcal{R}\phi$$

$$\mathcal{R} = \frac{\ell}{\mu A}, \quad \mu = \mu_r \mu_0, \quad \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

ℓ is the *mean* path, along the centre line of the core. Same flux in every segment of one loop, so B is largest where the core is thinnest. Reluctance is constant in air ($\mu_r = 1$) and not constant in steel.

magnetic

$$\text{MMF } \mathcal{F} = NI$$

$$\text{flux } \phi$$

$$\text{reluctance } \ell/(\mu A)$$

$$B = \phi/A, \quad H = \mathcal{F}/\ell$$

electric

$$\text{EMF } e$$

$$\text{current } i$$

$$\text{resistance } \ell/(\sigma A)$$

$$J = i/A, \quad E = e/\ell$$

Series reluctances add, parallel ones add as reciprocals; MMF drops round a loop equal the sources (KVL); fluxes into a junction sum to zero (KCL).

B-H, hysteresis and core loss

$B = \mu H$ bends because domains align until the core saturates: linear region, knee, saturation. Machines run **near the knee**. Under AC the core traces a hysteresis loop: residual B_r at $H = 0$, coercive H_c to bring B to zero; loop area is energy lost per cycle. Soft (narrow loop) for cores, hard (wide loop, large B_r) for permanent magnets. A smaller amplitude traces a nested minor loop.

ON THE FORMULA SHEET

$$P_h = k_h V B_{max}^n f \quad (n = 1.5-2.5), \quad P_e = k_e V (f t B_{max})^2$$

Doubling f doubles P_h and quadruples P_e ; thinner laminations (0.35–0.5 mm, varnish-insulated) cut only P_e . Leakage flux skips the core, so more MMF is needed. Fringing enlarges a gap's area, $A_g = (w + \ell_g)(d + \ell_g)$ or a stated percentage, so $B_g < B_{core}$; apply it to the gap only.

Solving a core

Split into segments, $\mathcal{R} = \ell/(\mu A)$ each, combine, $\phi = \mathcal{F}/\mathcal{R}$, $B = \phi/A$. A gap shortens its leg's iron by ℓ_g .

WORKED EXAMPLE · MODULE 3 · EXAMPLE 1 · PP.28--32

$N = 250$, $\mu_r = 3000$, depth 10 cm; no gap at 2 A. Mean paths 37.5, 35, 37.5, 35 cm:

$$\begin{aligned} \mathcal{R} &= 9947.184 \times 2 + 9284.038 + 6189.359 = 35\,367.765 \\ \phi &= \frac{500}{35\,367.765} = 0.014137 \text{ Wb}, \quad B = 1.414 \text{ T} \end{aligned}$$

With 0.5 mm gap, area +2.5%: $\mathcal{R}_g = 38\,818.279$, $\mathcal{R}'_2 = 9270.775$, $\mathcal{R} = 74\,172.781$, so $I = \mathcal{R}\phi/N = 4.195 \text{ A}$, about $2.1\times$.

WORKED EXAMPLE · MODULE 3 · EXAMPLE 2 · P.33 (P.36'S TOTAL IS WRONG)

Stator 185 cm, 428.36 cm^2 ; rotor 10 cm, 424.12 cm^2 ; two 0.5 mm gaps of 450 cm^2 ; $\mu_r = 2500$; $NI = 600$.

$$\mathcal{R} = 13\,747 + 750.5 + 2(8842) = 32\,182, \quad \phi = 18.64 \text{ mWb}, \quad B_g = 0.414 \text{ T}, \quad B_s = 0.435 \text{ T}$$

The flux crosses the gap *twice*; with one gap you get 25.7 mWb, the usual slip. The slide's own total, 24 090, counts $\mathcal{R}_r$ twice and $\mathcal{R}_g$ once, so its 0.0249 Wb is off too.

EMF and force

ON THE FORMULA SHEET

$$e = N \frac{d\phi}{dt} = \frac{d\lambda}{dt}, \quad \lambda = N\phi; \quad e = (\vec{v} \times \vec{B}) \cdot \vec{\ell} = vB\ell; \quad \vec{F} = i(\vec{\ell} \times \vec{B}), \quad F = i\ell B$$

Lenz: the induced current's flux opposes the *change* in flux. Example of ours: 100 turns, $\phi = 0.01 \sin 377t$ gives $e = 377 \cos 377t$ V.

Practice, with answers

Problem Set 3 is tutorial practice, not assessed. Try each before reading its answer.

1. (PS3 P1) $\mu_r = 800$, 500 turns, $\phi = 5$ mWb. Answer: $\mathcal{R} = 59\,683 + 72\,946 + 119\,366 = 251\,995$; $I = 2.52$ A; $B_{top} = 0.667$ T, $B_{right} = 2.0$ T.
2. (PS3 P2) Three legs, 300 turns at 1 A on the centre, $\mu_r = 1500$, gaps +5%. The text gives left 0.050 cm and right 0.070 cm, but the official solution follows the figure: left 0.07 cm, right 0.05 cm. Answer: $\mathcal{R}_L = 319\,720$, $\mathcal{R}_R = 276\,442$; $\phi_c = 1.468$ mWb, $\phi_L = 0.681$, $\phi_R = 0.787$ mWb; $B_{gL} = 0.185$ T, $B_{gR} = 0.214$ T (the official 0.215 divides the rounded 0.79). The smaller gap carries more.
3. (PS3 P3) $\mathcal{R} = 76\,630$; aiding $\phi = 6.52$ mWb, opposing 1.30 mWb. State which you read from the winding senses; the official solution reads aiding.
4. (PS3 P4) $H = 500/2.6 = 192.3$ A·t/m; the curve gives $B \approx 0.17$ T, $\phi = 3.83$ mWb, $\mu_r = 703.5$: the assumed μ_r of 1200 was not a good one.
5. (PS3 P5) 2 A up, 1 m, 0.5 T right: $F = 1$ N into the page.
6. (Ours) Mean path 60 cm, 16 cm^2 , $\mu_r = 2000$, 1 mm gap at +4%, 400 turns, 1.0 T in the core. Answer: $\mathcal{R}_m = 148\,959$, $\mathcal{R}_g = 478\,230$, $I = 2.51$ A, $B_g = 0.962$ T; without the gap 0.60 A.

Lesson source src/ece260_magnetic_circuits.jsx, 2026-10-03, PS3 P2 and P4 updated 2026-10-07; ECE 260 Module 3 slides (S. Azad, Fall 2026), Problem Set 3 and its official solutions, filed under lessons/ECE260/materials/. The course bans generative AI on assessments: this sheet is study material, never an answer to graded work.